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Asymptotic analysis of radiation extinction of stretched premixed flames

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Abstract

The flammability limit, radiation extinction of stretched premixed flame and effect of non-unity Lewis numbers are analyzed by the large-activation-energy asymptotic method. Particular attention is paid to the effect of Lewis number, the upstream and downstream radiation heat losses as well as the non-linearity of radiation. Explicit expressions for the flame temperature, extinction limit and flammability limit are obtained. The C-shaped extinction curve is reproduced. The dependence of radiation heat loss and the Lewis number effect on the stretch rate and flame separation distance is investigated. The effects of fuel Lewis number, oxidizer Lewis number, upstream radiation heat loss and the non-linearity of radiation on the C-shaped extinction curve are also examined. The results demonstrate a significant influence of these parameters on the radiation extinction and flammability limit and provide a good explanation to the experimental results and numerical simulations.

Keywords: Stretched premixed flame; Radiation extinction; Flammability limit; Asymptotic analysis

1. Introduction

The development of low NO_x and CO_2 emission combustors and fire safety control require accurate determination of the flammability limit. It is well known that radiation heat transfer is a dominant mechanism for near limit flames [1–7] and is characterized by a very strong non-linearity as well as spectral dependency [8]. A recent study by Ju et al. [8] using the statistical narrow-band model showed that a fundamental flammability limit exists for a gas mixture due to radiation heat loss via the difference between the spectral characteristics of reactant and product and temperature broadening of the emission spectra.

However, the flammability limit can be changed through the appearance of flame stretch and curvature. A series of experimental, numerical and theoretical studies [9–17] have made it clear that the flammability limit of stretched or curved flame below a critical Lewis number can be lower than the fundamental flammability limit. It was shown that the extended flammable region is due to the existence of the Near Stagnation plane Flame (NSF) and the flammability

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Nomenclature

A	constant defined in Eqs. (25)
$\bar{a}$	velocity gradient in the axial direction
B	constant defined in Eq. (23)
C	constant defined in Eq. (36)
$\bar{C}_p$	constant defined in Eq. (36)
D_a	Damköhler number, Eq. (5)
$\bar{E}$	activation energy
F	functions defined in Eqs. (10) and (23)
H_{if}	function defined in Eq. (7)
k	flow geometry (0: Cartesian; 1: axisymmetrical)
$\bar{k}_0$	frequency factor of reaction
L_{ei}	Lewis number of species i
M_i	molecular weight of species i
m	fraction of the downstream radiation heat loss in chemical heat release
n	parameter defined in Eq. (34)
$\bar{K}_P$	Planck mean absorption coefficient
p_0	parameter defined in Eq. (33)
$\bar{Q}$	chemical heat release per unit mass
R	universal gas constant
T	temperature normalized by $\bar{Q}/\bar{C}_p$
T_a	non-dimensional activation temperature, $\bar{E}\bar{C}_p/R\bar{Q}$
x	axial coordinate normalized by $\sqrt{\bar{\lambda}/\bar{\rho}\,\bar{C}_p\bar{a}(k+1)}$
y_F	fuel concentration at inner zone
Y_i	mass fraction of species i normalized by $v_i\bar{M}_i/v_F\bar{M}_F$
z	coordinate defined in Eq. (11)
$\bar{\rho}$	density
$\bar{\lambda}$	coefficient of heat conductivity
v_i	stoichiometric ratio
ω	reaction rate defined in Eq. (4)
ϵ	small parameter, T_f^2/T_a
ξ, τ, ζ	inner coordinate
θ	inner temperature
σ	Stefan–Boltzmann constant
Λ	reduced Damköhler number defined in Eq. (34)

Subscripts and superscripts

e	extinction
f	flame
F	fuel
i	species or temperature
O	oxidizer
r	radiation
0	outer solution
+, −	burned, unburned sides
∞	infinity

limit of the stretched flame below the critical Lewis number is characterized by the limit of the NSF [15–17]. The flammable region of the NSF is bounded by the stretch extinction limit at large stretch and the radiation extinction limit at low stretch. Numerical simulations [13–17] further demonstrated that multi-flame bifurcations are intrinsic phenomena of radiative stretched flames and the limits of these flame regimes are sensitive to the Lewis numbers of reactants and to both upstream and downstream radiation heat losses, particularly at low stretch. However, numerical simulations can not provide an explicit correlation between the extinction limits and the above individual parameters, which is very important for parameter selection in engine design and fire safety control. Therefore, to bring a clear understanding of the extinction mechanism and obtain an explicit correlation of the flammability limit, it is necessary to perform an analytical study to investigate the effects of Lewis numbers of both the fuel and oxidizer, the non-linearity of radiation, and the upstream and downstream radiation heat losses.

It has been shown that the activation energy asymptotic analysis [18,19] is an important way to understand the physics of the flame. Radiation effects on the extinction of stretched diffusion flames have been extensively studied by using this method [20–22]. However, analytical studies on radiative stretched premixed flames are relatively few, particularly when the non-linearity of radiation heat loss is taken into account. The first analysis of the radiative counterflow premixed flame is conducted by Sohrab and Law [23]. The analysis successfully examined the flame extinction by considering the non-linear radiation heat loss. Unfortunately, the flammability limit and radiation extinction at low stretch rate was not examined and the analysis was confined to unity Lewis number. Therefore, it is not clear to what extent the Lewis number effect and the radiation heat loss affect the flammability limit and the radiation extinction. The radiation extinction of counterflow premixed flame was later analyzed by Buckmaster [14]. The analysis successfully reproduced the C-shaped extinction curve [9–12] and showed four typical kinds of flame bifurcations. The analytical results agree qualitatively well with the results of an independent numerical study [13]. However, like the conventional treatment, the radiation heat loss is simplified by using a linearized heat loss term. Furthermore, the analysis is emphasized only on the fuel Lewis number below unity and the upstream radiation heat loss of flame front is neglected. Therefore, it is still unknown how the non-linearity of the radiation and the upstream radiation heat loss affect the flammability limit. In view of the above discussions, the purpose of the present study is to analyze the flammability limit and radiation extinction limit of the counterflow premixed flame using the asymptotic strategy [18,23] and to examine the effects of the Lewis number of the fuel and the oxidizer, upstream and downstream radiation heat loss as well as the non-linear characteristics of radiation heat loss. The present analysis is an extension of the studies of [18] and [23] and a complementation to the research work of [14].

2. Formulation

The axisymmetric counterflow premixed flame configuration is adopted here. Twin flames are formed near the stagnation plane of the two opposed mixture flows. Because of the symmetry, we only need to consider the solution in the region of [0, ∞] (see Fig. 1). Based on the constant thermal property assumption, the non-dimensional conservation equations for energy and species can be written as

$$x\frac{\mathrm{d}T}{\mathrm{d}x}+\frac{\mathrm{d}^2T}{\mathrm{d}x^2}=-D_{\mathrm{a}}\omega+q_{\mathrm{r}}$$

$$x\frac{\mathrm{d}Y_{\mathrm{F}}}{\mathrm{d}x}+\frac{1}{L_{\mathrm{eF}}}\frac{\mathrm{d}^2Y_{\mathrm{F}}}{\mathrm{d}x^2}=D_{\mathrm{a}}\omega$$

$$x\frac{\mathrm{d}Y_{\mathrm{O}}}{\mathrm{d}x}+\frac{1}{L_{\mathrm{eO}}}\frac{\mathrm{d}^2Y_{\mathrm{O}}}{\mathrm{d}x^2}=D_{\mathrm{a}}\omega \tag{1}$$

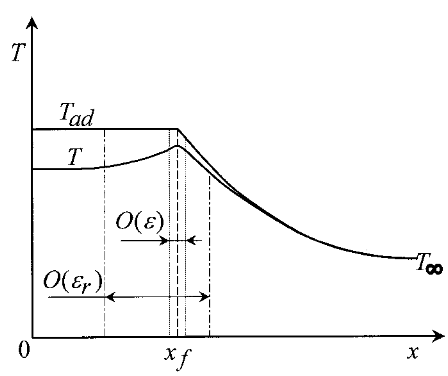

Fig. 1. The schematic model of the analysis.

with boundary conditions

$$x\to+\infty;\quad T=\bar{T}_\infty\bar{C}_{\mathrm{p}}/\bar{Q},\quad Y_{\mathrm{F}}=\bar{Y}_{\mathrm{F}\infty},$$

$$Y_{\mathrm{O}}=\bar{Y}_{\mathrm{O}\infty}\nu_{\mathrm{F}}\bar{M}_{\mathrm{F}}/\nu_{\mathrm{O}}\bar{M}_{\mathrm{O}}$$

$$x\to 0;\quad \mathrm{d}T/\mathrm{d}x=0,\quad \mathrm{d}Y_{\mathrm{F}}/\mathrm{d}x=0,\quad \mathrm{d}Y_{\mathrm{O}}/\mathrm{d}x=0. \tag{2}$$

Here the parameters with and without overbar denote the dimensional and non-dimensional variables, respectively. x is the axial coordinate normalized by $\sqrt{\bar{\lambda}/\bar{\rho}\bar{C}_{\mathrm{p}}\bar{a}(k+1)}$. T, Y_{F} and Y_{O} are, respectively, the non-dimensional temperature, fuel and oxidizer mass fractions normalized by $\bar{Q}/\bar{C}_{\mathrm{p}}$, 1 and $\nu_{\mathrm{O}}\bar{M}_{\mathrm{O}}/\nu_{\mathrm{F}}\bar{M}_{\mathrm{F}}$. $\bar{a}$ is the stretch rate and k is the flow configuration. $\bar{\rho}$ is the density. $\bar{C}_{\mathrm{p}}$, $\bar{\lambda}$ and $\bar{Q}$ are, respectively, the specific heat at constant pressure, thermal conductivity and chemical heat release per unit mass of fuel. ν_i and $\bar{M}_i$ denote the stoichiometric ratio and the molecular weight, respectively. Subscript '∞' represents the parameters at infinity.

On the right-hand side of Eq. (1), q_{r} is the non-dimensional volumetric heat loss and is approximated by the optically thin grey gas model for simplicity

$$q_{\mathrm{r}}=\frac{4\bar{K}_{\mathrm{p}}\sigma\bar{Q}^3(T^4-T_\infty^4)}{\bar{C}_{\mathrm{p}}^4\bar{\rho}\,\bar{a}(k+1)} \tag{3}$$

where σ and $\bar{K}_{\mathrm{p}}$ are, respectively, the Stefan–Boltzmann constant and the Planck mean absorption coefficient of the mixture. ω and D_{a} in Eq. (1) denote the non-dimensional reaction rate and the Damköhler number. By adopting the one-step irreversible reaction model, ω and D_{a} can be written as

$$\omega=Y_{\mathrm{F}}Y_{\mathrm{O}}\exp(-T_{\mathrm{a}}/T) \tag{4}$$

$$D_{\mathrm{a}}=\bar{k}_0\nu_{\mathrm{O}}\bar{\rho}/\bar{a}(k+1)\bar{M}_{\mathrm{F}}. \tag{5}$$

Here, T_{a} is the non-dimensional activation temperature and k_{o} is the frequency factor of the reaction.

3. Outer solution

In the limit of large activation energy, chemical reaction is limited to a very thin region at x_{f} (flame location, see Fig. 1) with a thickness of $O(\epsilon)$. ϵ is a small parameter defined as $T_{\mathrm{f}}^2/T_{\mathrm{a}}$ and T_{f} is the flame temperature. In addition, since the radiation heat loss depends on the Planck mean absorption coefficient and has a strong non-linearity of temperature, it is only important in a region around the thin reaction zone, where both the temperature and the concentration of the emitting gas are high. Thus, the solution domain can be divided into three regions: the inner reaction zone of $O(\epsilon)$, the radiation–convection–diffusion zone of $O(\epsilon_r)$ and the outer convection–diffusion zone. In this analysis, the total radiation heat loss is an $O(\epsilon)$ of the total chemical heat release and the Lewis numbers of the fuel and the oxidizer deviate from unity only by an $O(\epsilon)$.

In the convection–diffusion zone, the radiation term and reaction term in Eq. (1) can be omitted. The solution of temperature and species mass fractions can be obtained by integrating Eq. (1). At $x > x_f$, we have

$$T = T_\infty + \frac{T_f - T_\infty}{H_{Tf}} \int_\infty^x \exp(-\eta^2/2)\, d\eta$$

$$Y_F = Y_{F\infty} - \frac{Y_{F\infty}}{H_{Ff}} \int_\infty^x \exp(-L_{eF}\eta^2/2)\, d\eta$$

$$Y_O = Y_{O\infty} + \frac{Y_{Of} - Y_{O\infty}}{H_{Of}} \int_\infty^x \exp(-L_{eO}\eta^2/2)\, d\eta \qquad (6)$$

where

$$H_{if} = \int_0^{x_f} \exp(-L_{ei}\eta^2/2)\, d\eta - \sqrt{\frac{\pi}{2L_{ei}}}, \quad i = T, F, O; \quad L_{eT} = 1. \qquad (7)$$

At $x < x_f$, the solutions can be given as

$$T = T_0, \quad Y_F = Y_{F0}, \quad Y_O = Y_{O0} \qquad (8)$$

where 'f' in Eq. (6) denotes the variables at flame location and '0' in Eq. (8) denotes the outer zone solutions on the side of stagnation plane.

As mentioned earlier, it is assumed in the present analysis that the Lewis numbers of both the fuel and the oxidizer deviate from unity only by $O(\epsilon)$, a jump condition for fuel and oxidizer mass fraction can be obtained as

$$\frac{1}{L_{eF}}\left[\frac{dY_F}{dx}\right]_-^+ - \frac{1}{L_{eO}}\left[\frac{dY_O}{dx}\right]_-^+ = 0. \qquad (9)$$

Then we have

$$Y_{Of} = Y_{O\infty} - Y_{F\infty} F(L_{eF}, L_{eO}, x_f)$$

$$F(L_{eF}, L_{eO}, x_f) = \frac{L_{eO} H_{Of}}{L_{eF} H_{Ff}} \exp[x_f^2 (L_{eO} - L_{eF})/2]. \qquad (10)$$

4. Radiation zone

In the radiation–convection–diffusion zone, the chemical reaction can be omitted. By introducing

$$z = \sqrt{\frac{2}{\pi}} H_T(x) \qquad (11)$$

where $H_T(x)$ is defined in Eq. (7). The energy equation in Eq. (1) can now be written as

$$2\frac{d^2T}{dz^2} = -\pi D_a \exp(x^2)\omega + \pi \exp(x^2) q_r. \qquad (12)$$

Following the formulation of Sohrab and Law [23], the thickness of radiation zone can be considered as $O(\epsilon_r)$ with

$$1 \gg \epsilon_r = \frac{d(\ln T)}{d(\ln q_r)} \gg \epsilon. \qquad (13)$$

Therefore, the radiation heat loss and the temperature T can be expanded using ϵ_r

$$q_r = q_{rf} \exp[(T - T_f)/\epsilon_r T_f]$$

$$T = T_f + \epsilon_r \theta_r. \qquad (14)$$

Here q_{rf} is the radiation intensity at flame location and θ_r denote the derivation of temperature from the flame temperature due to radiation heat loss.

By employing an inner coordinate ξ_r in the radiation zone,

$$\xi_r = (z - z_f)/\epsilon_r. \qquad (15)$$

Eq. (12) can be rewritten as

$$2\frac{d^2\theta_r}{d\xi_r^2} = -\pi\epsilon_r D_a \exp(x_f^2)\omega + \pi\epsilon_r q_{rf} \exp(x_f^2 + \theta_r/T_f). \qquad (16)$$

By integrating the above equation and matching with the outer solution, we can obtain

$$\left[\left(\frac{d\theta_r}{d\xi_r}\right)^2\right]_{+\infty}^{-} = q_r^-, \quad \left[\left(\frac{d\theta_r}{d\xi_r}\right)^2\right]_{-\infty}^{+} = q_r^+. \qquad (17)$$

Here '+' and '−', respectively, denote the location of the burned and unburned sides of the flame front $q_r^\pm$ and the corresponding heat fluxes are

$$q_r^\pm = \pi\epsilon_r \exp(x_f^2) q_{rf}^\pm T_f. \qquad (18)$$

5. Reaction zone

In the reaction zone, the convection term and the radiation heat loss are very small compared with the diffusion and reaction terms. Thus the energy equation and the fuel conservation equation can be written as

$$\frac{d^2 T}{dx^2} = -D_a\omega$$

$$\frac{1}{L_{eF}}\frac{d^2 Y_F}{dx^2} = D_a\omega. \tag{19}$$

A jump condition can be obtained as

$$\left[\frac{dT}{dx}\right]_-^+ + \frac{1}{L_{eF}}\left[\frac{dY_F}{dx}\right]_-^+ = 0. \tag{20}$$

By using the outer solution given in Eq. (6) and $Y_{Ff}=0$ (the present study deals with fuel lean flames), the jump condition of heat flux can be given as

$$\left[\frac{dT}{dx}\right]_-^+ = -\frac{Y_{F\infty}}{L_{eF}H_{Ff}}\exp(-L_{eF}x_f^2/2). \tag{21}$$

Combining Eqs. (17) and (21) and using outer solutions, a relation between the chemical heat release, the radiation heat loss and the conduction heat loss can be obtained as

$$Y_{F\infty}F(L_{eF}, x_f) = \sqrt{\frac{B^+T_f(T_f^4 - T_\infty^4)}{\bar{a}}} + \sqrt{\frac{B^-T_f(T_f^4 - T_\infty^4)}{\bar{a}} + (T_f - T_\infty)^2} \tag{22}$$

where

$$F(L_{eF}, x_f) = \frac{H_{Tf}}{L_{eF}H_{Ff}}\exp[-(L_{eF}-1)x_f^2/2]$$

$$B^{\pm} = H_{Tf}^2\,\exp(x_f^2)A_r, \quad A_r = \frac{8\epsilon_r\bar{K}_p\sigma\bar{Q}^3}{\bar{C}_p^4\bar{\rho}(k+1)}. \tag{23}$$

$Y_{F\infty}$ and $F(L_{eF},\ x_f)$ on the left-hand side of Eq. (22), respectively, denote the temperature increases due to chemical heat release and the Lewis number effect. The first and second terms on the right-hand side respectively represent the heat flux induced by the downstream radiation heat loss and the sum of the heat fluxes induced by upstream radiation heat loss and thermal conduction on the unburned side. The adiabatic flame temperature can be obtained by setting the radiation heat loss to zero in Eq. (22). Moreover, Eq. (22) reduces to the result of [23] if the Lewis number effect is neglected and the radiation heat loss is assumed to be independent of the stretch rate and temperature distribution. In order to close the equations, we still need to correlate the Damköhler number with the flame temperature.

Since our interest here is the lean flammability limit, the fuel is always the insufficient species. By expanding the temperature and mass fraction of fuel in the reaction zone using the small parameter ϵ,

$$T = T_f - \epsilon\theta, \quad Y_F = \epsilon y_F \tag{24}$$

and using an inner coordinate

$$\xi = (x - x_f)/\epsilon A, \quad A = -\frac{L_{eF}H_{Ff}}{Y_{F\infty}}\exp(L_{eF}x_f^2/2) \tag{25}$$

the energy equation can be rewritten as

$$\frac{d^2\theta}{d\xi^2} = D_a\epsilon A^2\omega$$

$$\frac{1}{L_{eF}}\frac{d^2 y_F}{d\xi^2} = D_a\epsilon A^2\omega \tag{26}$$

with boundary condition

$$\left.\frac{d\theta}{d\xi}\right|_{\xi\to-\infty} = -m, \quad \left.\frac{d\theta}{d\xi}\right|_{\xi\to+\infty} = 1 - m. \tag{27}$$

Here m denotes the fraction of the downstream radiation heat loss of the chemical heat release

$$m = \frac{\sqrt{B^+T_f(T_f^4 - T_\infty^4)/\bar{a}}}{Y_{F\infty}F(L_{eF}, x_f)}. \tag{28}$$

By integrating Eq. (26) and matching with outer solution, we can obtain a relation between the temperature and fuel concentration

$$y_F = (\theta + m\xi)L_{eF}. \tag{29}$$

Substituting Eq. (29) to Eq. (26) and expanding the reaction source term, the energy equation is reduced to

$$\frac{d^2\theta}{d\xi^2} = \Lambda(m\xi + \theta)\exp(-\theta) \tag{30}$$

where

$$\Lambda = D_a\epsilon^2A^2L_{eF}Y_{Of}\,\exp(-T_a/T_f). \tag{31}$$

Eq. (30) and boundary condition (27) can be further rewritten in the Linán's equation form

$$2\frac{d^2\tau}{d\zeta^2} = \tau\,\exp(-m\zeta - \tau)$$

$$\left.\frac{d\tau}{d\zeta}\right|_{\zeta\to-\infty} = 1, \quad \left.\frac{d\tau}{d\zeta}\right|_{\zeta\to+\infty} = 0 \tag{32}$$

using the transformation

$$\zeta = -\xi - p_0/m, \quad \tau = \theta - m\zeta - p_0, \quad p_0 = \ln(2\Lambda). \tag{33}$$

By matching the inner temperature solution of Eq. (32) with the outer solution, the relation between the Damköhler number and the flame temperature as well as the radiation heat loss can be obtained as

$$\Lambda = \exp(-mn)/2, \quad n = \lim_{\zeta \to -\infty}(\tau + \zeta). \tag{34}$$

Eq. (32) has been examined by Linán [18] and an approximate expression of nm as a function of m is given as

$-0.2 < m < 0.5$:

$$nm = 1.344m - 4m^2(1-m)/(1-2m) + 3m^3 - \ln(1-4m^2)$$

$$-0.2 > m: \quad nm = -\ln(0.6307m^2 - 1.344m + 1). \tag{35}$$

Combining Eq. (34) with Eq. (31), the final form of the Damköhler number (stretch rate) can be given as

$$\frac{1}{\bar{a}} = \frac{C}{H_{\mathrm{Ff}}^2 \exp(L_{\mathrm{eF}} x_{\mathrm{f}}^2)} \frac{1}{Y_{\mathrm{Of}}} \left(\frac{T_{\mathrm{a}}}{T_{\mathrm{f}}^2}\right)^2 \exp\left(\frac{T_{\mathrm{a}}}{T_{\mathrm{f}}} - nm\right)$$

$$C = \frac{\bar{M}_{\mathrm{F}}(k+1)\bar{Y}_{\mathrm{F}\infty}^2}{2\bar{k}_0 \nu_{\mathrm{O}} \bar{\rho} L_{\mathrm{eF}}^3}. \tag{36}$$

It can be easily proven that Eq. (36) again reduces to the result of [23] if the Lewis numbers of fuel and oxidizer are unity.

6. Results and discussions

The solution of flame temperature as a function of stretch rate can be obtained directly by using Eqs. (36), (35), (28), (22) and (10). Our goal here is to examine the radiation extinction limit and the flammability limit. It is known that both the stretch extinction and the radiation extinction occur at $x_{\mathrm{f}} \to \epsilon$ [13–17]. Therefore, by taking the limit of $x_{\mathrm{f}} \to 0$ and using the fact that m is small at extinction limit, Eq. (36) reduces to

$$\frac{1}{\bar{a}_{\mathrm{e}}} = \frac{2CL_{\mathrm{eF}}}{\pi(Y_{\mathrm{O}\infty} - Y_{\mathrm{F}\infty}\sqrt{L_{\mathrm{eO}}/L_{\mathrm{eF}}})} \left(\frac{T_{\mathrm{a}}}{T_{\mathrm{f}}^2}\right)^2 \exp\left(\frac{T_{\mathrm{a}}}{T_{\mathrm{f}}}\right). \tag{37}$$

The left-hand side of the above equation is a hyperbolic function of $\bar{a}_{\mathrm{e}}$ and the right-hand side is an exponential function of $\bar{a}_{\mathrm{e}}$ $[T_{\mathrm{f}} = T_{\mathrm{f}}(\bar{a}_{\mathrm{e}})]$. Therefore, Eq. (37) has two solutions when the fuel concentration is high enough. The two solutions are respectively the stretch extinction limit and the radiation extinction limit. As the fuel concentration decreases, these two solutions approach each other and the resulting merging point is the flammability limit of the radiative stretched flame. Eq. (37) can be solved directly by using Newton iteration and it converges so quickly that only several limited steps are required. In the limiting case of zero upstream radiation heat loss, the expression of the flame temperature at the flammability limit can be given as

$$\frac{\pi}{2} = B^+ T_{\mathrm{f}}^5 \left(2 + \frac{T_{\mathrm{a}}}{T_{\mathrm{f}}^2}\right)^2 \frac{CL_{\mathrm{eF}}}{(Y_{\mathrm{O}\infty} - Y_{\mathrm{F}\infty}\sqrt{L_{\mathrm{eO}}/L_{\mathrm{eF}}})T_{\mathrm{f}}^2} \left(\frac{T_{\mathrm{a}}}{T_{\mathrm{f}}^2}\right)^2 \exp\left(\frac{T_{\mathrm{a}}}{T_{\mathrm{f}}}\right). \tag{38}$$

The above equation clearly shows that a larger radiation heat loss or fuel Lewis number results in flame extinction at a larger flame temperature.

In the following calculations, the thermal and chemical parameters are chosen according to a typical hydrocarbon fuel/air mixture. They are $\bar{Q} = 3.8 \times 10^7$ J/kg, $\bar{C}_{\mathrm{p}} = 1400$ J/kg K, $\bar{k}_0 = 1.3 \times 10^{11}$ m^3/mol/s, $\bar{M}_{\mathrm{F}} = 0.016$ kg/m^3, $\bar{\rho} = 1.0$ kg/m^3, $T_{\mathrm{a}} = 0.9$, $\bar{K}_{\mathrm{p}} = 2.3$ m−1, $\bar{T}_\infty = 300$ K, $\nu_{\mathrm{O}} = 2$ and $k = 1$. The variation of $F(L_{\mathrm{eF}}, L_{\mathrm{eO}}, x_{\mathrm{f}})$ and $F(L_{\mathrm{eF}}, x_{\mathrm{f}})$ with the location of flame front, which represent the Lewis number effect on the local fuel and oxidizer concentrations, is plotted in Fig. 2. It can be seen that though the Lewis numbers deviate from unity by only 0.1, the Lewis number effect is considerable. The results also show that the Lewis number effect becomes negligible when the flame is far away from the stagnation plane. However, the Lewis number effect plays an increasing role in affecting the local fuel and oxidizer concentrations as the

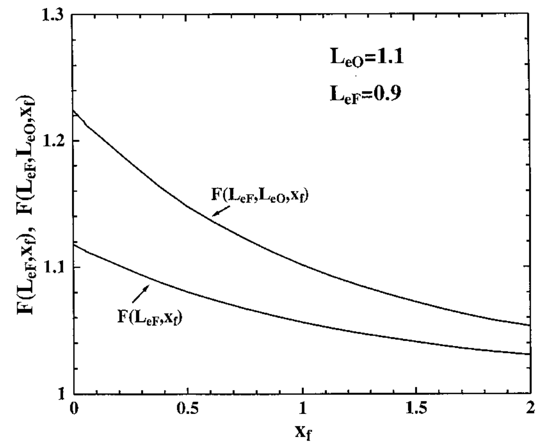

Fig. 2. The dependence of the Lewis number effect on flame location.

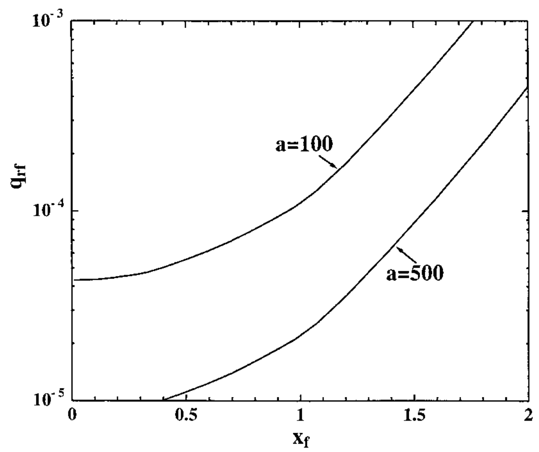

Fig. 3. The dependence of radiation heat loss on flame location and stretch.

flame moves towards the stagnation plane. This result agrees with the previous experimental observations and numerical calculation [13].

Fig. 3 shows the non-dimensional radiation heat loss as a function of flame location at stretch rate of 100 s^{-1} and 500 s^{-1}. It can be seen that radiation heat loss increases dramatically as the flame separation distance increases. This is because the downstream burned gas volume increases and results in an increase of the total radiation heat loss. In addition, as the stretch rate goes down, the radiation heat loss also increases. This implies that the total radiation heat loss strongly depends on the residence time of the emitting gases. This result gives a clear explanation why the low stretched flame suffers larger influence of radiation heat loss. In the calculation of [23], the radiation heat loss term is set to a constant. Therefore, the radiation

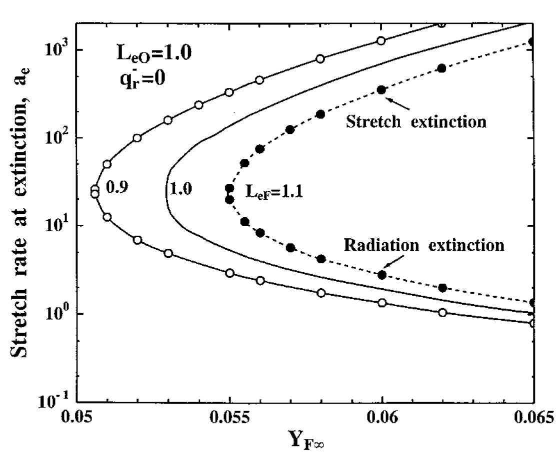

Fig. 4. The effect of fuel Lewis number on flammability limit and radiation extinction.

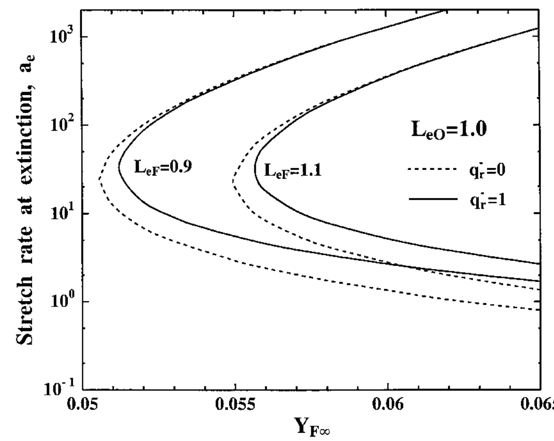

Fig. 5. The effect of upstream radiation heat loss on the flammability limit and radiation extinction.

heat loss depends neither on the flame location nor on the stretch rate. As a result, the radiation extinction limit at low stretch rate and the flammability limit cannot be predicted.

The effects of fuel Lewis number on the flammability limit and the extinction limit are shown in Fig. 4. The oxidizer Lewis number is unity and the upstream radiation heat loss is set to zero here. Fig. 4 shows that for a given fuel concentration, there are two extinction limits. The upper one is the stretch extinction limit and the lower is the radiation extinction limit. At the upper extinction limit an excessive stretch quenches the flame due to incomplete combustion. While at the lower extinction limit a very low stretch also quenches the flame due to large residence time of the emitting gas. Thus, flame can only exist at a moderate stretch rate at which the Lewis number effect enhances the flame strength. It can be seen that as the fuel Lewis number decreases, the stretch extinction shifts to a larger stretch rate and the radiation extinction limit moves to a lower stretch rate. As a result, the flammability limit decreases. This effect can be easily understood from Eq. (36). Eq. (36) shows that the lower the Lewis number, the higher the adiabatic flame temperature and thus the flame can survive larger radiation heat loss.

Fig. 4 also shows that even for Lewis number larger than unity, the C-shaped extinction curve still exists. This result supports the conclusion of the numerical study [15]. However, the flammable region becomes narrow and the flammability limit increases which is again in agreement with the detailed numerical calculation [15].

The effect of the upstream heat loss on the flammability limit is shown in Fig. 5. It can be seen that the upstream radiation heat loss has only negligible effect on the stretch extinction. This is because the preheat zone is so thin that the upstream radiation heat loss is

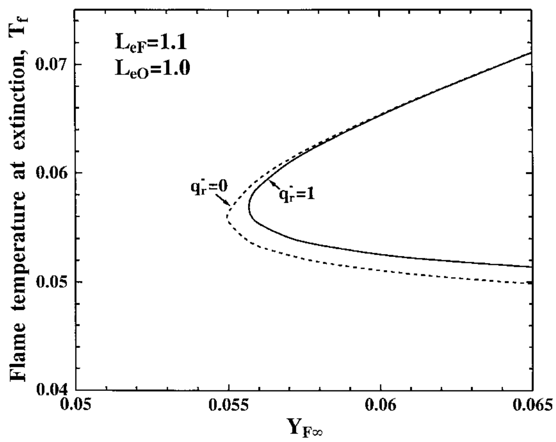

Fig. 6. The effect of upstream radiation heat loss on flame temperature at extinction.

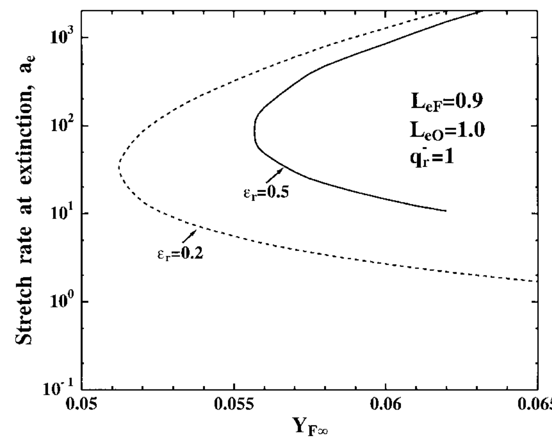

Fig. 8. The effect of non-linearity of radiation on the extinction limit.

very small compared with the chemical heat release. However, the upstream radiation heat loss has a considerable effect on both the flammability limit and the radiation extinction limit. Therefore, the neglect of the upstream radiation heat loss results in an over-prediction of the flammable region. The corresponding temperature curves with and without upstream radiation heat loss are plotted in Fig. 6. It can be easily understood that the reason why the flammable region narrows is because the flame is subjected to larger heat loss and thus quenches at a larger temperature.

The effect of the oxidizer Lewis number is shown in Fig. 7. It can be seen that the oxidizer Lewis number has a much smaller effect on the flammability limit compared with the fuel Lewis number. Therefore, an equal Lewis number approximation of the reactants is appropriate for the analysis of the extinction of the stretched premixed flame.

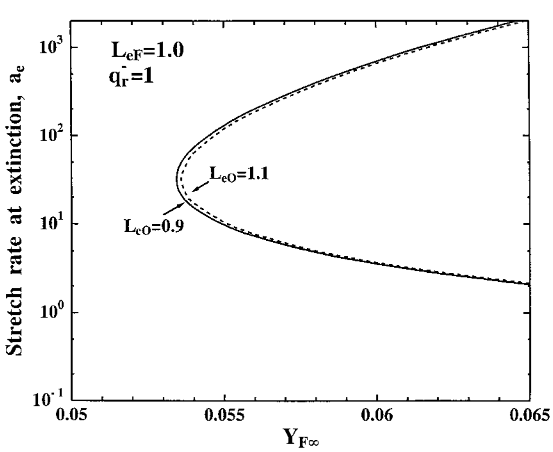

Fig. 7. The effect of oxidizer Lewis number on extinction limit.

Finally, the effect of the non-linearity of the radiation is shown in Fig. 8. $\epsilon_r = 0.2$ and 0.5, respectively, correspond to the nonlinear radiation heat loss of Eq. (3) and a linearized heat loss. In these two cases, the peak radiation heat losses at the flame front are assumed the same. It can be seen that the non-linearity of the radiation heat loss has a great impact on both the stretch extinction limit and the radiation extinction limit and the use of a linearized heat loss results in a larger flammability limit and a narrow flammable region. It should be noted that the effect of the non-linearity of the radiation heat loss on the stretch extinction for a real flame is not as large as that shown in Fig. 8 because the flame subjects to a much larger heat loss via the incomplete combustion. However, at the radiation extinction limit, the radiation heat loss is dominant. Therefore, the non-linearity of radiation is important to radiation extinction.

7. Conclusion

The flammability limit and radiation extinction of stretched premixed flame with radiation heat loss and non-unity Lewis number are analyzed by the large activation energy asymptotic method. Explicit expressions for the stretch and radiation extinction limits as well as the flammability limit are obtained and the C-shaped extinction curve is reproduced. It is shown that radiation heat loss strongly depends on flame separation and stretch rate. A low stretch rate can quench the flame by radiation heat loss. The results showed that the C-shaped extinction curve still exists at a fuel Lewis number larger than unity. It is also shown that the fuel Lewis number has a much larger impact on the flammable region than the oxidizer Lewis number.

A decrease of the fuel Lewis number dramatically extends the flammability limit. In addition, the results also show that the upstream radiation heat loss is important and cannot be neglected in the prediction of the radiation extinction limit. Moreover, the results indicate that the non-linearity of the radiation heat loss has a great impact on the flammability limit and the radiation extinction. The linearized approximation of radiation heat loss may result in under prediction of the flammability limit.

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